Describing Function Analysis of the Electric Nonlinear Model of a SRM Autonomous AC Generator

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Abstract—**This paper focuses on the Switched Reluctance Motor (SRM) used as self-excited generator. The phase inductance resonates autonomously with an external capacitor parallel connected producing an AC voltage almost sinusoidal. Although the voltage quality is not good enough to be directly connected the network utility it is able to be used as a battery charger in isolated locations. As the machine shows a nonlinear relationship among current, flux and angular position, the system equations are nonlinear therefore, analytical solutions describing its behaviour can not be found. However, it is shown that the system follows a known nonlinear second order equation describing mechanical systems. This experience is transferred to gain insight in this application. It is shown that the steady state can be identified as a limit cycle whose main variables are derived. The analysis procedure followed is known as the describing function method. It is an extended version of the frequency response method, and can be used to approximately analyze and predict nonlinear behaviour. The main use of describing function method is for the prediction of limit cycles in nonlinear systems. A simulated example is developed to illustrate the method and to derive the relationship among the electrical variables, useful for design purposes.**

I. INTRODUCTION

The Switched Reluctance Motor (SRM) can be used as a generator for braking purposes but the current is difficult to manage. That is the reason because it is not used for generating purposes. However, contrary to its common working manner where DC current is imposed in a phase, it can work with AC current naturally developed by a parallel connected resonant circuit formed by the phase inductance itself, an external capacitor and an AC load. Resonance is achieved for a rotor speed close to natural resonance of the circuit. Starting of resonance is built up by the remnant magnetic field of the phase's magnetic circuit. Under resonant conditions the circuit variables grow up oscillating and ending in a dynamic steady state. This state is determined by the rotating speed, the load resistance and the resonant parameters. Although the machine can be studied on a single phase basis, Switched

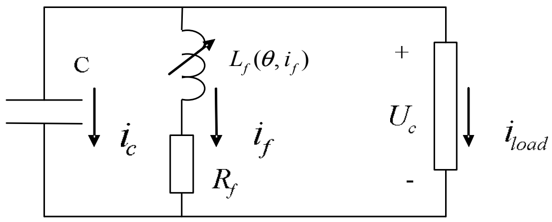

Figure 1. Single phase model of parallel connected circuit

Reluctance Machines working as three-phase autonomous generator has been studied for differences in behaviour when the phases are star-connected with and without neutral [1]. Although in synchronous three-phase generators start-connected without neutral reject third harmonic and provides better voltage wave-shape, in Autonomous Switched Reluctance Generators (ASRG) worsens its functioning because third harmonic forms part of its intrinsic nonlinear behaviour. Besides, the resonant circuit is serially connected. In our case, the parallel connection shown in Fig. 1 is preferred because load modifications can be easily achieved. Similarities between ASRG and electronic oscillators have been pointed out in [2]. Hill's equation describing parametric oscillators and its derived Mathieu's equation, in case of low resistor load, help in system understanding. Hill's equation is a linear differential equation with a periodic time varying parameter. This study sets the maximum value of load resistance as a function of working frequency and both maximum and minimum value of phase inductance. As the information provided is not enough to cope with specifications for design purposes, we propose in this paper a new approach to relate electrical parameters used in specifications. In this paper the equations of the single electric model are firstly derived. After stating the basics of the describing function method, the limit cycle of the nonlinear system is exploited to set the relationship among the electric variables. Finally, a set of specified data is run in a simulation model showing the limit cycle obtained.

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II. CIRCUIT EQUATIONS

Fig. 1 shows the circuit elements of the electric model. The phase inductance $L_f(\theta, i_f)$ shows a periodic dependence on rotor position and phase current. Current dependency is due to iron saturation and is responsible of its nonlinear behavior. The capacitor C and the load resistance Rc is externally connected. The Kirchhoff current law provides the following equations:

$$U_c - R_f \cdot i_f = \frac{d\Phi(\theta, i_f)}{dt} \qquad (1a)$$

$$\Phi(\theta, i_f) = L_f(\theta, i_f) \cdot i_f \qquad (1b)$$

$$C\frac{dU_c}{dt} = -(i_f + i_{load}) \qquad (1c)$$

Where linked flux $\Phi(\theta, i_f)$ and inductance $L_f(\theta, i_f)$ are nonlinear with current i_f and periodic with position rotor position θ. Letting RL to be the load resistance, $i_{load} = U_c / RL$. When the Laplace transform is applied to (1c):

$$-\left(\frac{RL}{C \cdot RL \cdot s + 1} + R_f\right) i_f = s \cdot \Phi \qquad (2a)$$

$$\begin{aligned} &-(RL + R_f + C \cdot RL \cdot R_f \cdot s) i_f = \\ &C \cdot RL \cdot s^2 \cdot \Phi + s \cdot \Phi \end{aligned} \qquad (2b)$$

The flux dependence on variables has explicitly been removed. Flux and current are assumed to be nonlinearly related by an odd function:

$$i_f = k_1 \cdot \Phi \cdot (1 + k_2 \cdot \Phi^2) \qquad (3)$$

The inductance definition leads to:

$$L_f(\theta, i_f) = \frac{\Phi(\theta, i_f)}{i_f} = \frac{1}{k_1 \cdot (1 + k_2 \cdot \Phi^2)} \qquad (4)$$

When the current is low, flux is low and the inductance is periodic in θ and non-saturated, thus it is not dependent on current and it is only dependent on position. (4) is approximated by the low order harmonic as:

$$\begin{aligned} &L_f(\theta) = L_f(\theta, i_f)\big|_{i_f \to 0} = \\ &= \frac{L_{max} + L_{min}}{2} - \frac{L_{max} - L_{min}}{2}\cos(2 \cdot \omega \cdot t) = \frac{1}{k_1(\theta)} \end{aligned} \qquad (5)$$

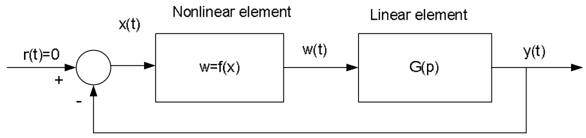

Figure 2 Split of a nonlinear system into linear and nonlinear blocks.

Substituting $k_1(\theta)$ in (3) and taking into account that $k_1(\theta)$ is also periodic:

$$i_f = (L_a + L_b \cdot \cos(2 \cdot \omega \cdot t))\Phi \cdot (1 + k_2 \cdot \Phi^2) \qquad (6)$$

Where:

$$L_a = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{L_m - (L_{max} - L_{min})\cos\theta} d\theta \qquad (7)$$

$$L_b = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{1}{L_m - (L_{max} - L_{min})\cos\theta} \cos\theta \cdot d\theta \qquad (8)$$

$$L_m = \frac{L_{max} + L_{min}}{2} \qquad (9)$$

Substituting (6) in (2b), it is obtained the nonlinear second order equation:

$$\begin{aligned} &-(RL + R_f + C \cdot RL \cdot R_f \cdot s)(L_a + L_b \cdot \cos(2 \cdot \omega \cdot t)) \cdot \Phi \cdot \\ &\cdot (1 + k_2 \cdot \Phi^2) = C \cdot RL \cdot s^2 \cdot \Phi + s \cdot \Phi \end{aligned} \qquad (10)$$

In (4) appears a term:

$$k_{att} = \frac{1}{1 + k_2 \cdot \Phi_2} \qquad (11)$$

This represents the attenuation factor of inductance due to flux amplitude. k_2 can be obtained from (4) by measuring phase inductance for maximum flux (or phase current) under aligned rotor position.

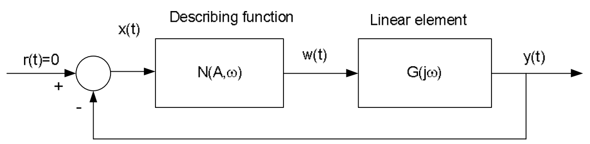

Figure 3 Nonlinear block representing the describing function.

The next step is handling (10) by using the describing function analysis.

III. DESCRIBING FUNCTION APPLIED TO DETECT A LIMIT CYCLE

Any system which can be transformed into the configuration in Fig. 2 can be studied using describing functions [3]. Hard nonlinearities as motor saturation identify systems that can be transformed like Fig.2.

Rearranging terms in (10), results in (12):

$$\begin{aligned}&-(RL+R_f)\cdot L_a\cdot k_2\cdot\Phi^3-(RL\cdot R_f\cdot C\cdot L_a\cdot k_2)\cdot s\,(\Phi^3)\\&-(RL+R_f)\cdot L_b\cdot\cos(2\cdot\omega\cdot t)\cdot\Phi\cdot(1+k_2\cdot\Phi^2)\\&-(RL\cdot R_f\cdot C)\cdot s\,(L_b\cdot\cos(2\cdot\omega\cdot t)\cdot\Phi\cdot(1+k_2\cdot\Phi^2))=\\&=(RL\cdot C)\cdot s^2\,\Phi+(1+RL\cdot R_f\cdot C\cdot L_a)\cdot s\,\Phi+(RL+R_f)\cdot L_a\cdot\Phi\end{aligned}\tag{12}$$

The idea underlying the method is based on the fact that if a limit cycle exists the forms of the signals are approximately sinusoidal. Due to the presence of the nonlinear block its output contains harmonics which will be filtered out by the linear block.

Linear block in Fig. 3 should have low pass properties. Therefore, it can be assumed that the forms in the whole system are basically sinusoidal. The left side in (13) contains the nonlinear terms and the right side is the linear block. The linear block is a second order system with low pass properties.

Let us now assume that there exist a self-sustained oscillation of amplitude A and frequency ω as shown in Fig. 3. The variables in the loop must satisfy.

$$\begin{aligned}&x=-y\\&w=N(A,\omega)\cdot x\\&y=G(j\omega)\cdot\omega\end{aligned}$$

Therefore, we have $y=G(j)\cdot N(A,\omega)\cdot(-y)$. Thus, the loop gain should be -1. Consequently, if limit cycle exist, the following equation should be met:

$$G(j\omega)\cdot N(A,\omega)+1=0\tag{13}$$

IV. CALCULATION OF N(A,ω)

The left hand side terms in (12) represents the nonlinear operations to be performed on flux. The result is applied to the linear system represented in the right hand side. Thus, the output of left hand side represents ω(t). The procedure assumes a sinusoidal input to the nonlinear element (terms of left-hand side in (11)) of amplitude A and frequency ω, i.e. $x(t)=A\cdot\sin(\omega\cdot t)$ The output ω(t) is of the form $\omega(t)=M\cdot\sin(\omega\cdot t+\theta)$ .Thus,

$$N(A,\omega)=\frac{M\cdot e^{j(\omega t+\theta)}}{A\cdot e^{j(\omega t)}}=\frac{M}{A}\cdot e^{j\theta}=\frac{(a+j\cdot b)}{A}\tag{14}$$

Letting the left-hand side in (12) to be $FNL(\Phi,\omega)$. The first harmonic response of $FNL(\Phi,\omega)$ to $\Phi=A\cdot\sin(\omega\cdot t)$ is obtained as:

$$\begin{aligned}a&=\frac{\omega}{\pi}\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}}FNL(A\sin(\omega\cdot t),\omega)\cdot\sin(\omega\cdot t)\cdot dt\\b&=\frac{\omega}{\pi}\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}}FNL(A\sin(\omega\cdot t),\omega)\cdot\cos(\omega\cdot t)\cdot dt\end{aligned}\tag{15}$$

Where from (12):

$$\begin{aligned}&FNL(.)=F1(.)+F2(.)+F3(.)+F4(.)\\&F1(.)=-(RL+R_f)\cdot L_a\cdot k_2\cdot(.)^3\\&F2(.)=-(RL\cdot R_f\cdot C\cdot L_a\cdot k_2)\cdot s(.)^3\\&F3(.)=-(Rc+RL)\cdot L_b\cdot\cos(2\cdot\omega\cdot t)\cdot F5(.)\\&F4(.)=-(RL\cdot R_f\cdot C)\cdot s(L_b\cdot\cos(2\cdot\omega\cdot t)\cdot F5(.))\\&F5(.)=(.)\cdot(1+k_2\cdot(.)^2)\end{aligned}\tag{16}$$

From (15) and (16) results:

$$\begin{aligned}a&=-\omega\cdot A\cdot RL\cdot R_f\cdot C\cdot\left[\frac{3}{4}\cdot L_a\cdot k_2\cdot A^2-\frac{L_b}{2}(1+k_2\cdot A^2)\right]\\b&=-A\cdot(RL+R_f)\cdot\left[\frac{3}{4}\cdot L_a\cdot k_2\cdot A^2-\frac{L_b}{2}(1+k_2\cdot A^2)\right]\end{aligned}\tag{17}$$

By substitution of (17) in (14), $N(A,\omega)$ is obtained. From the right-hand side of (11), the linear transfer function block $FL(\omega)$ is obtained:

$$FL(\omega) = \\ = \frac{1}{RL \cdot C \cdot s^2 + (1 + L_a \cdot RL \cdot R_f \cdot C) \cdot s + (RL + R_f) \cdot L_a} \quad (18)$$

The natural frequency of the limit cycle will be approximately:

$$\omega_0 = \sqrt{\frac{(RL + R_f) \cdot L_a}{RL \cdot C}} \quad (19)$$

The damped frequency of the limit cycle will be approximately:

$$\boxed{\omega_n = \omega_0 \sqrt{1 - \xi^2}} \quad (20)$$

With

$$\boxed{\xi = \frac{\omega_0}{2} \frac{1 + L_a \cdot RL \cdot R_f \cdot C}{(RL + R_f) \cdot L_a}} \quad (21)$$

By substitution of ω_0 in (13) and solving for A taking into account that the result must be real, it yields the flux amplitude in the limit cycle:

$$A = \sqrt{\frac{2 \cdot L_b}{k_2 \cdot (3 \cdot L_a - 2 \cdot L_b)}} \quad (22)$$

V. SIMULATION RESULTS

To test the proposed procedure an example is simulated using the following data: $C = 1mF$, $RL = 31\Omega$, $R_f = 1\Omega$, $L_{\max} = 280mH$, $L_{\min} = 40mH$. The following expressions can be derived from data: $L_m = 160mH$, $L_a = 9.45H^{-1}$ and $L_b = 8.53H^{-1}$. From (11), $k_2 = 0.01 \cdot Wb^{-2}$.

To test the analytic equations obtained, a simulation model of circuit in Fig. 1 has been developed as follows:

$$\begin{aligned} \dot{v}_c &= -\frac{v_c}{RL \cdot C} - \frac{\phi}{C \cdot L(t,\phi)} \\ \dot{\phi} &= v_c - R_f \cdot \frac{\phi}{L(t,\phi)} \end{aligned} \quad (23)$$

Where v_c is the capacitor voltage and ϕ is the linked flux.

The inductance $L(t,\phi)$ is derived from (4), (5) and (11):

$$\boxed{L(t,\phi) = L_f(\theta = \omega t) \cdot k_{att}(\phi)} \quad (24)$$

The generator has 3 pole pairs and it is running at $n = 291 \cdot rpm$. The predicted angular frequency by (20) is $98.7 \cdot rad/s$ against $91.4 \cdot rad/s$ obtained in the simulator. The predicted flux amplitude obtained by (22) is $12.3 \cdot Wb$ against $4.5 \cdot Wb$ obtained by the simulator.

Fig. 4 shows the linked flux versus phase current. The trajectory is scanned clockwise. The enclosed area represents the mechanical energy transformed into electrical energy.

Fig. 5 shows the phase portrait of the two state variables phase linked flux and capacitor voltage. This voltage is applied to the resistor load.

Fig. 6 shows the capacitor voltage and the phase inductance. The wave shape deformation indicates the presence of harmonics. It is worth to notice that capacitor

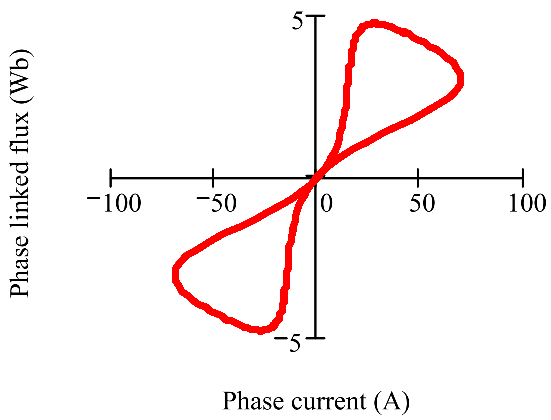

Figure 4 Phase-current trajectory followed clockwise by the machine. The enclosed area represents the mechanical energy transformed into electrical energy

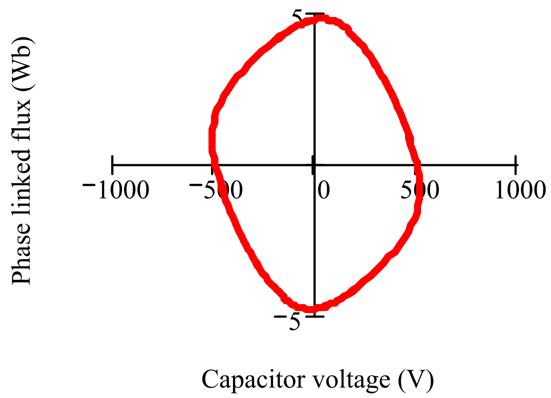

Figure 5 Phase portrait of limit cycle.

TABLE I.
HARMONICS IN CAPACITOR VOLTAGE

Order h	Amplitude (VRMS)	V(h)/V(1)
1	308.4	1
2	-	-
3	50.3	0.162
4	-	-
5	13.9	0.042
6	-	-
7	4.5	0.013
THD=16.8%		

voltage oscillates at half frequency of inductance.

The harmonic contents are in Table 1. As the curve shows shift symmetry, it only contains odd harmonics.

The first harmonic of capacitor voltage is 308Vrms and the THD is 16.8%. This wave does not have enough quality to be connected to the main. However, it can be used to non-critical loads like a battery bank in isolated sites. It can be loaded through a low cost rectifier. The ratio between phase current and load current is 3. The power developed in the load resistance is 3kW.

VI. CONCLUSSIONS

Switched reluctance machines can be used like self excited generators driving AC currents instead of DC ones, as usual. It has been shown that phase current and capacitor voltage oscillates at a frequency half of the phase inductance. The circuit equations are nonlinear due to saturation and inductance periodicity with rotor position. This makes difficult understanding the machine behaviour. The equations exhibit a limit cycle whose characteristic parameters were analyzed using describing functions. Analytical formulations were derived describing relationships among the electric variables. The formulations were applied to a simulated example.

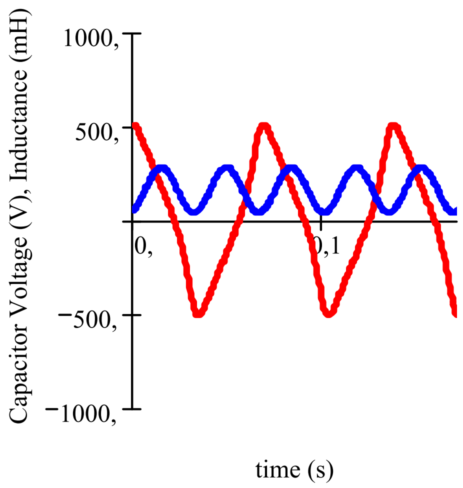

Figure 6 Capacitor voltage (V) in red and phase inductance (mH) in blue. Both are periodic but inductance oscillates at double frequency.

ACKNOWLEDGMENT

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